SCOTCH COLLEGE

Year 12 Mathematics Specialist

TEST 2 – Vectors

DATE: 8th March 2016 Name___________________________

Reading Time: 3 minutes

SECTION ONE: CALCULATOR FREE

TOTAL: 25 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet.

WORKING TIME: 25 minutes (maximum)

SECTION TWO: CALCULATOR ASSUMED

TOTAL: 28 marks

EQUIPMENT: Pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing instruments, templates, up to 3 Calculators,

1 A4 page of notes (one side only), SCSA formula sheet.

WORKING TIME: 25 minutes (minimum)

SECTION 1 Question	Marks available	Marks awarded	SECTION 2 Question	Marks available	Marks awarded
1	5		6	9	
2	6		7	7	
3	4		8	12	
4	6				
5	4				
Total	25			28	

Section One: Calculator-free **[25 marks]**

This section has **five (5)** questions. Answer **all** questions. Write your answers in the spaces provided

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Question 1 [5 marks]

A straight line passes through the points $P(2,-3)$ and $Q(5,3)$.

(a) Find the vector equation of the line in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [2]

(b) Find the equation of the line through P and Q in parametric form. [1]

(c) Find the equation of the line through P and Q in Cartesian form. [2]

Question 2 [6 marks]

The point A lies on the line with equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(2\mathbf{i} - \mathbf{j})$ and the point B has position vector $4\mathbf{i} - 5\mathbf{j}$. Use a method involving a dot product to determine the position vector of A so that the distance from A to B is a minimum. [6]

Question 3 [4 marks]

Point A has position vector $\begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}$ and point B has position vector $\begin{pmatrix} 6 \\ 5 \\ -6 \end{pmatrix}$. Find the position vector of the point P that divides AB internally in the ratio $2:3$.

Question 4 [6 marks]

(a) Find a vector perpendicular to the two vectors:

$\overrightarrow{OP} = \vec{i} - 3\vec{j} + 2\vec{k}$

$\overrightarrow{OQ} = -2\vec{i} + \vec{j} - \vec{k}$ [3]

(b) If $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are position vectors for the points P and Q, use your answer to part (a), or otherwise, to find the area of the triangle OPQ. [3]

Question 5 [4 marks]

Points P and Q have coordinates $(3, 1, -2)$ and $(4, 2, -1)$ respectively.

(a) Write a vector equation for the line passing through P and Q. [2]

(b) Show that the vector $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ is perpendicular to the line through P and Q. [1]

(c) Write down a vector equation of the plane containing P and Q with $2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ as its normal vector. [1]

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Section Two: Calculator-assumed **[25 marks]**

This section has **three (3)** questions. Answer **all** questions. Write your answers in the spaces provided

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Question 6 [9 marks]

Two rockets are fired from different positions at the same time. Rocket 1 leaves from position $-7\boldsymbol{i}+9\boldsymbol{j}-5\boldsymbol{k}$ km at a velocity of $5\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k}$ km/min and Rocket 2 leaves from position $-6\boldsymbol{i}-5\boldsymbol{j}+2\boldsymbol{k}$ km at a velocity of $9\boldsymbol{i}+6\boldsymbol{j}-3\boldsymbol{k}$ km/min. Each rocket leaves a trail of smoke and, although the rockets do not collide, their smoke trails do intersect.

(a) Find the coordinates of the point at which the smoke trails intersect. [4]

(b) Find the position of Rocket 1 three minutes after firing. [1]

(c) Find the shortest distance of Rocket 1 from the smoke trail of Rocket 2, three minutes after firing. Give your answer to the nearest metre. [4]

Question 7 [7 marks]

(a) The equation of a sphere is given by $x^2 + y^2 + z^2 - 6x + 4y + 8z = 153$. Determine the vector equation of the sphere. [3]

(b) Determine the position vector(s) of the points of intersection between the sphere and the line $\boldsymbol{r} = -3\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k} + \lambda(-2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k})$. [4]

Question 8 [12 marks]

Let $\boldsymbol{r} = \begin{pmatrix} 2t+5 \\ -2t-1 \\ t \end{pmatrix}$, $t \in R$, be an equation of line L.

The plane P has a normal vector $\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ and passes through the point $A(-1, 0, 4)$.

(a) Show that the point $B(9, -5, 2)$ lies on the line L. [2]

(b) Give the normal vector equation of the plane P. [2]

(c) Find the shortest distance that plane P is from the origin. [2]

(d) Show that the line L meets the plane P at the point $C\,(1, 3, -2)$. [3]

(e) Find the angle between the line L and the plane P. (Give your answer correct to 1 decimal place.) [3]

END OF QUESTIONS